Biological Physics II - Tutorial

SS 2019
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No tutorial this friday (31.05.19), but a 2 hour lecture from Prof. Dr. Berenike Maier.
The week after, we have a 2 hour tutorial.

Week 7-8 - Gillespie algorithm and Eigenvalues of ODEs-part 2

A. Gillespie algorithm

We will try to numerically implement the Gillespie algorithm from the lecture describing a birth-death process (for example, mRNA molecules). The rate of change of probability of having l mRNA molecules is given by:

$$\frac{dP(l)}{dt} = \beta_s P(l-1) - \beta_s P(l) + k_d(l+1)P(l+1) - k_d l P(l)$$

1) explain all the terms appearing in this equation.

2) The total rate of events (birth and death) is $\beta_{tot} = \beta_s + k_d l$. Perform the Gillespie algorithm as following:

- Choose an initial number of mRNA molecules larger or equal to 0, as well as β_s and k_d constant and positive.
- Draw the waiting time τ until a next event happens (it may be birth or death) from an exponential distribution $P(\tau) = \beta_{tot} e^{-\beta_{tot}\tau}$, using for example in MATLAB the function: `exprnd`$(\beta_{tot}, 1, 1)$.
- Draw a random number n between 0 and 1. If n is smaller then $\frac{\beta_s}{\beta_{tot}}$, one mRNA molecule is synthesized. If it is larger, one is degraded.
- Update the number of mRNA molecule l and afterwards the total rate.
- Repeat the last three steps N times (large enough to reach steady state). Plot the number of mRNA molecules l against the total elapsed time $\sum \tau$ after each event.
- Repeat the whole algorithm for different values of k_d and β_s. What fixed point is reached ?

3) BONUS: Prove that the waiting time is indeed drawn from an exponential distribution if the synthesis and degradation of a molecule are both drawn from a Poisson distribution.

B. Number of chromosomal DNA

During the life cycle of a cell, and because of the replication of chromosomal DNA, there is a probability that the cell possesses two copies of a gene of interest. If we imagine that for a fraction f of the cell cycle, there is one copy of the gene (corresponding to an average number λ of mRNA), and for a fraction $(1-f)$, there are two copies of the gene (corresponding to an average number 2λ of mRNA), the probability for a certain number m of mRNA molecules is given by

$$p(m) = f\frac{\lambda^m e^{-\lambda}}{m!} + (1-f)\frac{(2\lambda)e^{-2\lambda}}{m!} \quad .$$

1) Find the average number $\langle m \rangle$ of mRNA molecules.

2) In a similar fashion, find $\langle m^2 \rangle$.

3) Determine the *Fano* factor F, defined as the ratio between the variance and the mean value. Plot F as a function of f for different values of λ. Interpret the result.

C. (BONUS) Linearisation and Eigenvalues of an ODE - part 2

Consider the following system of differential equations:

$$\dot{x} = 2x - y - x^2$$
$$\dot{y} = x - 2y + y^2$$

1) Linearise the system and determine the Jacobian matrix. Then determine the Eigenvalues for each fixed point. What does the nature of the Eigenvalues tell you about the latter?

Good luck! :) If you have questions regarding the exercises, feel free to contact me.