Biological Physics II - Tutorial

SS 2019
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Week 6 - Partitioning of molecules and Eigenvalues of ODEs-part 2

A. Partitioning of molecules

Fluorescent molecules inside a cells body make it possible to observe it under any microscope equipped to capture this fluorescence. After division of the cell, the number of fluorophores N is distributed to the 2 daughter cells with numbers n_1 and $N - n_1$. The number of ways we can divide N undistinguishable molecules into two cells is given by

$$W(n_1) = \frac{N!}{n_1!(N - n_1)!} \quad .$$

1) Prove that the standard deviation of molecules n_1 partitioned to one of the daughter cells is given by $\langle n_1^2 \rangle - \langle n_1 \rangle^2 = Npq$, where p is the probability a molecule will go into daughter cell 1, and q is the probability the molecule will go into cell 2. To obtain this result, use the properties of the binomial distribution (hint: https://en.wikipedia.org/wiki/Binomial_distribution#Expectation

2) The intensity I recorded by the camera of the microscope is assumed to be proportional to the number of fluorophores, with proportionality constant α. Show that the average squared difference of intensities can be expressed as $\langle (I_1 - I_2)^2 \rangle = \alpha I_{tot}$. For this, use $p = q = 0.5$ and $I_{tot} = I_1 + I_2$.

3) Use stochastic simulations (take a constant α for all simulations) to validate the result from 2). Proceed as following: Choose N, then generate N random numbers between 0 and 1. If a given random number is < 0.5, assign it to daughter cell 1 (in the variable n_1), if it is larger, assign it to daughter cell 2. Compute the intensities I_1, I_2, I_{tot}, and repeat 100 times, thus getting 100 3-tuples. Then choose a different N and repeat the whole process. At the end, plot the root-mean-square difference of intensities vs. the total intensity and compare it with the analytical result.

B. Linearisation and Eigenvalues of an ODE - part 2

Consider the following system of differential equations:

$$\dot{x} = 2x - y - x^2$$
$$\dot{y} = x - 2y + y^2$$

1) Linearise the system and determine the Jacobian matrix. Then determine the Eigenvalues for each fixed point. What does the nature of the Eigenvalues tell you about the latter?

Good luck! :)