Biological Physics II - Tutorial

SS 2019
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There will be no tutorial on the 10th of May 2019, but a two-hour lecture from Prof. Dr. Maier. On the 17th of May, there will be no lecture, but a 2 hour tutorial.

Week 4-5 - Fritz Hugh Nagumo Model, Membrane Potential and Eigenvalues of ODEs

A. The Fritz-Hugh-Nagumo Model

Consider the simplified Fritz-Hugh-Nagumo model from the lecture:

$$\begin{aligned}\dot{v} &= f(v) - w + I_a \\ \dot{w} &= bv - \gamma w \\ f(v) &= v(a-v)(v-1)\end{aligned}$$

1) Describe and explain all the terms appearing in this set of differential equations. Explain how the simplification from the general model treated in the lecture takes place.

2) For $I_a = 0$, plot the nullclines analytically (in a w vs. v plot). How many fixed points could there be when the parameters a, b, γ are varied? Which ones are stable?

3) For $I_a = 0$, solve the system numerically and plot the pairs $(v(t), w(t))$ (not the fixed points, but the dynamic) for different initial variables v_0 and w_0 (choose them wisely, and let the system relax so that the system becomes stationary) in a plot with axis v and w. Choose $a = 0.25$ and $b = \gamma = 0.002$. Which fixed point is reached? What should a, b, γ be to relax into the other fixed points?

4) For $a = 0.25$, $b = \gamma = 0.002$, and now $I_a = 0.5$, plot again the dynamics of the system for different initial variables. What do you observe?

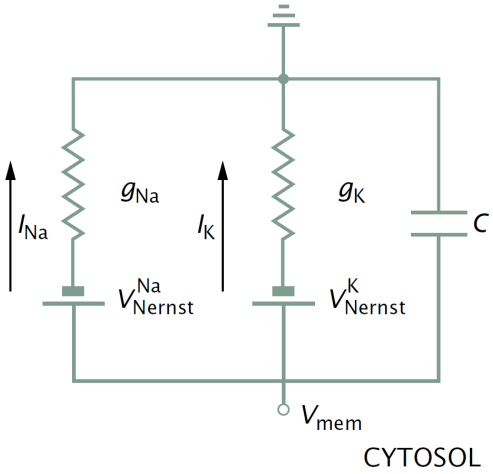

Figure 1. From Phillips et al., Physical Biology of the Cell: Mapping of a cell membrane onto an equivalent electrical circuit.

B. The membrane as an electrical circuit

As depicted in figure 1, the cell membrane can be modelled as an electrical circuit whose total membrane potential is related through:

$$I = g(V_{mem} - V_{Nernst})$$

This corresponds to an equivalent electrical circuit with only one resistance and battery in series.

1) Try to understand the circuits! explain the analogy with the biological situation. What does each parameter (I, g, V) represent? Why is their a current?

2) Write down the relation between the current and the voltage for each channel (Sodium and Potassium) of the circuit. What is the total current? What is the total conductance?

3) Solve the set of equations for V_{Nernst}.

C. Linearization and Eigenvalues of an ODE

Consider the Verhulst-Pearl equation (logistic equation) describing population growth:

$$\dot{x} = rx(1 - x)$$

with the initial condition $x(t = 0) = x_0$ and some parameter r.

1) What are the fixed points of this equation? Write down the Taylor expansion around

the fixed points in a general manner. What is the Jacobian of the equation?

2) Determine the eigenvalues of the linearised equation. Solve the linearised equation for both Eigenvalues. Which fixed point is stable, which one is unstable?

Good luck!