Biological Physics II - Tutorial

SS 2019
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Week 2 - Molecular Biology and Theoretical Mechanics

A: Molecular Biology

The following is a special case of the model for the two cross-repressing genes discussed in the lecture:

$$\dot{x} = h(y) - w_x x,$$
$$\dot{y} = h(x) - w_y y$$

with $h(x) = \frac{1}{1+x^2}$ and $h(y) = \frac{1}{1+y^2}$. Unless specified otherwise, use $w_x = w_y = 0.1$. Don't forget to use logarithmic scales for plotting where appropriate.
1) Try to understand the model, and briefly describe the role of each term.
2) Integrate the model numerically for some initial conditions. Describe and interpret your observations.
3) Sketch a bifurcation diagram as follows:

- plot w_x on the x-axis, ranging from 0.0 to 0.3 ($w_y = 0.1$).
- the y-axis shows x or y, respectively.
- For appropriately distributed values of w_x, simulate the model for several random initial conditions.
- For each simulation, add two points to the sketch showing the values of x and y after 1000 timesteps.

Briefly interpret the result. Where do you suspect the bifurcations to occur?
4) The *nullclines* of a system are the curves of points for which the time derivative of *one* of the dynamical variables is zero. Plot the nullclines (analytical calculations) in a graph with axis x and y for one of the representative value of each of the three parameter regimes separated by the bifurcations.
5) BONUS: What effect will noise have on the system? try the following equations:

$$dx = (h(y) - w_x x)dt + \sigma x dW_1$$
$$dy = (h(x) - w_y y)dt + \sigma y dW_1$$

where $W_{1,2}$ are independent standard Wiener processes ($\sigma = 0.25$, see the lecture for how to incorporate the noise).

B: Theoretical Mechanics

A dynamical system $\dot{x} = f(x)$ can be classified according to the sign of $\nabla \cdot f$, i.e. the divergence of the right-hand-side of an ODE. Systems with $\nabla \cdot f < 0$ are called dissipative, systems with $\nabla \cdot f = 0$ are called conservative. To simplify things at first, assume the divergence is constant. Answer the following questions:
1) Can conservative systems have attracting fixed points or repellors?
2) Can conservative systems have limit cycles?
3) What do systems with $\nabla \cdot f > 0$ look like? Are they important?
4) What kind of systems have you encountered most in your studies of physics so far? What kind of dynamical systems do you expect to be most relevant in biological applications? Why?

Good luck! :)