Biological Physics II - Tutorial

SS 2019
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Week 11 - One component Turing

A. Turing Instability with one Morphogen

Last week, we saw that the different diffusion of two morphogens can destabilize a steady state and lead to spatially periodic patterns (for small times). We now want to analyse this situation in the presence of only one morphogen. For N cells arranged in a ring (with position index r), the discrete reaction diffusion equation for the morphogen concentration Y reads:

$$\dot{Y}_r = D \cdot (Y_{r+1} + Y_{r-1} - 2Y_r) + g(Y_r) \quad ,$$

where g now describes the chemical reactions involving the morphogen within a cell.

1) For a periodic perturbation from the steady state, i.e. $Y_r = Y^* + y(t)e^{i\frac{2\pi r}{\lambda}}$, show by linearisation of the above equation that the amplitude $y(t)$ of the perturbation evolves in time as $\dot{y}(t) = \left[B - 4D\sin^2(\frac{\pi}{\lambda})\right] y(t)$, with $B = \frac{dg}{dy}\Big|_{Y^*}$.

2) From the periodicity of the ring cell arrangement ($Y_r = Y_{r+N}$), what are the allowed values of the wavelength λ for the perturbation?

3) Derive the conditions under which the uniform steady state is unstable to the given periodic perturbations. Argue that this one-component Turing system does not lead to spatially periodic pattern of morphogen concentration (differentiate two cases: a) system is stable in the absence of diffusion, and b) system is unstable in the absence of diffusion).

Good luck :)